

Estimating the Hausman test for multilevel Rasch model

Kingsley E. Agho
School of Public health
Faculty of Medicine
The University of Sydney

James A. Athanasou
Faculty of Education
University of Technology, Sydney

Kingsley Agho
School of Public Health
Room 305, Edward Ford Building (A27)
The University of Sydney
NSW 2006, AUSTRALIA
Phone: +61 2 9351 4384
Fax: +61 2 9351 5049
Email:kingsleya@health.usyd.edu.au

Word count (incl abstract + formulae + references) = 3980

Estimating the Hausman test for multilevel Rasch model

Abstract

In recent times, there has been an increased interest in the use of the multilevel Rasch model in medical, educational and psychological testing. In the Rasch multilevel model, the first level entails item responses, which rely on item variation and person ability; the second level describes variation and covariation between person ability within school and the third level describes variation and covariation between schools. While, the Hausman test for the multilevel Rasch random effect with intercept is the shape differences in item difficulty parameter estimates from the two and three level Rasch method. In this study, the Hausman test for the multilevel Rasch model was presented using data set from the second International Association for the Evaluation of Educational Achievement (IAEA) mathematics study for high school pupils in Australia. The sample consisted of 10 dichotomously-scored items from 50 students drawn from two stage sampling and only year 9 students with completed information were used for the analysis. The main findings were that the Hausman test suggested a statistical difference in item difficulty estimates between two and three-level Rasch model and ignoring the effect of clustering will result in biases when interpreting item difficulty parameter in Rasch measurement model.

Key words: Multilevel; Rasch model and Hausman test

Introduction

Since the introduction of the Rasch measurement model to educational and psychological testing the two most commonly used fit statistics for Rasch are the likelihood ratio test and the chi-square test (Traub & Wolfe, 1981). The drawbacks of these two fit statistics test for Rasch are that the asymptotic properties of the chi-square tests of fit cannot be determined mathematically and Andersen's maximum likelihood theory is not applicable to joint estimation (see Traub & Wolfe, 1981).

The two major challenges in fit statistics for Rasch are lack of fit due to sampling variability across different levels of hierarchy and lack of fit due to using an inappropriate model for examining fit statistics for Rasch. In these two methods, the most daunting task is the inappropriate use of the specification model (Skrondal & Rabe-Hesketh, 2004). Another criticism levied against the likelihood ratio tests are that it is not a well known model fit statistic for nested or multilevel models (Skrondal & Rabe-Hesketh, 2004). This view was supported by Berger and Selike (1987) who argued that significant probabilities and evidence are often in conflict for two nested models and conditioning on a single selected model uncertainty may lead to underestimation of standard errors (Miller, 1984).

The drawbacks of the Andersen's likelihood ratio test both for the Rasch model (see Traub & Wolfe, 1981 for details) and the multilevel Rasch led to the introduction of the Hausman specification test (Hausman, 1978) in this paper. The Hausman test is easy to implement and potentially useful because the test requires only the estimated covariance matrix of the two item difficulty estimates for Rasch (see the middle term in equation 10). The Hausman test also follows the general theory of maximum likelihood estimation by estimating standard errors and the test statistics (see equation 10).

Importantly, there has been a strong and growing interest among Rasch users to understand the effect of clustering when estimating item difficulty in Rasch. However, the Rasch measurement estimates that ignore clustering can lead to the misinterpreting of

the difficulty parameter of an item. It will be demonstrated later that ignoring clustering can result in biases when interpreting item difficulty estimates in the multilevel Rasch model.

In this paper, we first describe the similarity of the fixed-effect Rasch model (also known as one-level Rasch model) and multilevel Rasch model. Secondly, we present the Hausman specification test for the multilevel Rasch model. Thirdly, we describe the purpose of the study. Finally, an application of the Hausman specification test to the multilevel Rasch model is presented with discussion by using a data set from the second International Association for the Evaluation of Educational Achievement (IAEA) mathematics study for high school pupils in Australia conducted in 1978. The collected sample was based on two-stage sampling procedure which is similar to three-level model in Rasch.

Two and three-level Rasch model

In this section, we will follow similar path employed by Roberts and Herrington (2005) to show the equivalent of the fixed-effect Rasch model (Rasch, 1960) and the multilevel Rasch model with random intercepts. We start-out by introducing the Rasch one-parameter logistic model for dichotomous items. Let X_{ig} be the binary or dichotomous (1,0) response for person g ($g= 1, \dots, G$) and item i ($i = 1, \dots, N$), where 1 denotes a correct answering and 0 denotes an incorrect answering. Let $P_{ig} = P(X_{ig} = 1)$ and $Q_{ig} = 1 - P_{ig} = P(X_{ig} = 0)$. Then one-level Rasch model is of the form:

$$P_{ig}(\theta) = \frac{\exp(\theta_g - \alpha_i)}{1 + \exp(\theta_g - \alpha_i)} \quad (1)$$

$$Q_{ig}(\theta) = \frac{1}{1 + \exp(\theta_g - \alpha_i)} \quad (2)$$

The logistic one-level Rasch model (η_{ig}) is of the form:

$$\eta_{ig} = \log \left(\frac{P_{ig}(\theta)}{Q_{ig}(\theta)} \right) = \theta_g - \alpha_i \quad (3)$$

where θ_g is the person's ability and α_i is the item difficulty parameter. The expression in equation (3) is the one-level logistic Rasch model which entails item responses that rely on item variation and person ability. Then the two level-Rasch model can be written as:

$$P_{qig}(\theta) = \frac{\exp(\gamma_{00} + \mu_{0g}) - \alpha_q}{1 + \exp(\gamma_{00} + \mu_{0g}) - \alpha_q} \quad (4)$$

$$Q_{qig}(\theta) = \frac{1}{1 + \exp(\gamma_{00} + \mu_{0g}) - \alpha_q} \quad (5)$$

The logistic two-level Rasch model is of the form:

$$\eta_{ig} = \log \left(\frac{P_{qig}(\theta)}{Q_{qig}(\theta)} \right) = (\gamma_{00} + \mu_{0g}) - \alpha_q \quad (6)$$

where,

$$\text{Person ability} = \beta_{0g} = \gamma_{00} + \mu_{0g} = \theta_g$$

$$\alpha_{qg} = \alpha_q, q = 1, \dots, K-1 \text{ and } K = 10$$

Item difficulty = 0 for the reference item

$$\text{Item difficulty} = -\alpha_q$$

The intercept β_{0g} is related to a person g 's latent trait estimate while the slopes α_{qg} are related to the Rasch item difficulty estimates. Where X_{qig} is a q th dummy variable for person g assigned to a value of “-1” q when represent item i and a value of “0” otherwise. Then the coefficient for the intercept α_{0g} is now the expected effect from dropping items (called reference item).

In the three-level Rasch model, the first level entails item responses, which rely on item variation and person ability. The second level describes variation and covariation between person ability within school. The third level describes variation and covariation between schools. Following this, Roberts and Herrington, (2005) expressed the three-level Rasch model as:

$$P_{qigm}(\theta) = \frac{\exp(\gamma_{000} + \mu_{00m} + \eta_{0gm}) - \alpha_q}{1 + \exp(\gamma_{000} + \mu_{00m} + \eta_{0gm}) - \alpha_q} \quad (7)$$

$$Q_{qigm}(\theta) = \frac{1}{1 + \exp(\gamma_{000} + \mu_{00m} + \eta_{0gm}) - \alpha_q} \quad (8)$$

The logistic three-level Rasch model is of the form:

$$\eta_{ig} = \log\left(\frac{P_{qigm}(\theta)}{Q_{qigm}(\theta)}\right) = (\gamma_{000} + \mu_{00m} + \eta_{0gm}) - \alpha_q \quad (9)$$

where,

$$\text{Person ability} = \beta_{0gm} = \gamma_{000} + \mu_{00m} + \beta_{0gm} = \theta_g$$

$$\alpha_{qgm} = \alpha_{qg} = \alpha_q$$

Item difficulty = 0 for the reference item

$$\text{Item difficulty} = -\alpha_q, q = 1, \dots, K-1$$

The expressions in (6) and (9) are similar to the Rasch model described in (3). Relating to the expressions in (6) and (9), Raudenbush, Johnson and Sampson (2003) have argued that multilevel Rasch model with random effect treat each item to have a fixed measure and each person to be a random representative of a distribution – that is, items with extreme scores are not used for estimating person measures but persons with extreme scores are used in the estimation of item parameters (Mike Linacre, 2006, personnel email) this approach may depart from the classical fixed-effect Rasch model (see, Rasch, 1960; Linacre, 2001). The fixed effect Rasch conceptualised each person and each item to have a fixed measure which implies, persons with extreme scores are not used for estimating item difficulties and items with extreme scores are not used for estimating person measures (Mike Linacre, 2006, personnel email).

Hausman test application to Multilevel Rasch model

The Hausman test for Rasch is closely related to Andersen's likelihood ratio test for Rasch. The null hypothesis of the Hausman test is the same as the likelihood Ratio test, Wald test and Lagrange Multiplier test and they have the same asymptotic power for local alternatives (Fisher & Molenaar, 1995; Weesie, 1999). A related logistic form for the two-level Rasch model random effect with intercept is described in (6) while that of the three-level Rasch model random effect with intercept is in (9). Thus, the

corresponding Hausman specification test (H_n) for multilevel Rasch model random effect with intercept is given by:

$$H_n = \hat{q}_n^T \left[\hat{V}ar(\hat{\alpha}_{q_2}) - \hat{V}ar(\hat{\alpha}_{q_3}) \right]^{-1} \hat{q}_n \quad (10)$$

where $\hat{\alpha}_{q_2}$ is the item difficulty parameter of two-level Rasch model and $\hat{\alpha}_{q_3}$ is the item difficulty parameter of three-level Rasch model. The Hausman test equations in (10) have asymptotically a null $\chi^2(k)$ distribution where k is the length of difficulty parameter $\hat{\alpha}_{q_2}$. $\hat{V}ar(\hat{\alpha}_{q_2})$ and $\hat{V}ar(\hat{\alpha}_{q_3})$ are the asymptotic variance of $\hat{\alpha}_{q_2}$ and $\hat{\alpha}_{q_3}$. \hat{q}_n^T is the transpose of \hat{q}_n and, define $\hat{q}_n = [\hat{\alpha}_{q_2} - \hat{\alpha}_{q_3}]$ where \hat{q}_n the estimated difference between the two-level difficulty parameter and three-level item difficulty parameter.

The multilevel Rasch model with random effect has been studied widely in measurement theory (Raudenbush, et al., 2003; Robert & Herrington, 2005). To the best of our knowledge, only a few papers (Robert & Herrington, 2005) have provided a detailed practical solution to the multilevel Rasch measurement model. The main aim of this current paper is to investigate the effect of clustering in estimating the Rasch item difficulty and the statistical difference between variations of item difficulty estimates for two - and three- multilevel Rasch methods provide practical solutions to multilevel Rasch model using the Hausman test.

Method

Participants

The participants in the study comprised 50 students (male=26, female = 24) ranging in age from 12.5 to 13 years (mean = 12.98 years, SD = 0.14) from the IAEA mathematics study (Rosier, 1980 a,b) for high school pupils in Australia. The sample of the population was drawn in two stages. Firstly, secondary schools were selected at random with a probability proportional to the number of 13-year-old students. Secondly, a group of 25

students was selected at random from each of the sample schools. In this paper, only year 9 students with completed information were used in the analysis. The sample used in this study consisted of three types of schools that is, 52% comprehensive; 13% selective academic and 22% selective vocational.

Instrument

The items used in the study consisted of 10 dichotomously scored items – a copy of the test is provided in the Appendix. The instrument was administered in schools by teachers and the main aim of the instrument was to compare curriculum changes in mathematics between 1964 and 1978 in Australia. The 10 questions in the Appendix were recoded as binary items (1 for correct, 0 for wrong). The technical dataset was purchased from the Australian Social Sciences Data Archive.

Analysis

In setting up the data set to run the “MASS” function in the R statistical package, the data was reshaped lengthwise (see Agho & Athanasou, 2005 for an illustration) and the variable “response” was created and the response variable represent whether or not person g receive a score of “1” or “0” on item i . The person identification (id) number for the second-level unit is included when performing two-level analysis while the school id number is included for the third-level units and the command “glmmPQL” for estimating the hierarchical generalised linear models is used. (item 1 is the reference items see, equations (6) and (9)). The analysis was carried out using R statistical computing (available at www.r-project.org). The intra-item, correlation coefficients were estimated by dividing the variance of the item by the total variance (that is, variance of the items plus variance of persons plus variance of schools (which is, $0.001+1.099+0.789$ in Table 2). The intra-person correlation coefficient is the variance of the person divided by total variance while the intra-school correlation coefficient is the variance of the school divided by total variance. To estimate the Hausman specification test, the expression in (10) is applied following from equations (6) and (9).

Results

Table 1 reports the mean, category and frequency of items used in the final study. The item means ranged between 0.28 – 0.96. From the table, item 3 has the highest proportion of correct items with the mean of 0.96.

Table 1

Standard deviation, mean and item response for 1978 IAEA mathematics study (N=50)

Items	Mean	standard deviation	Frequency	
			0=incorrect	1 = correct
item 1	0.82	0.39	9	41
Item 2	0.94	0.24	3	47
Item 3	0.96	0.37	2	48
Item 4	0.84	0.37	8	42
Item 5	0.68	0.47	16	34
Item 6	0.66	0.48	17	33
Item 7	0.28	0.45	36	14
Item 8	0.42	0.50	29	21
Item 9	0.68	0.47	16	34
Item 10	0.54	0.50	23	27

Table 2 present the comparison for 3-level and the fixed-effect Rasch model for a dichotomously scored item. The second column in Table 2 gives the item difficulty estimate of the random-effect 3-level Rasch model while column 6 gives the item difficulty estimate for a fixed-effect Rasch model. At the fixed-effect Rasch model, we observe that two items (items 7 and 8) were difficult for the sample for two methods. In general, ignoring clustering will lead to bias when interpreting item difficulty for Rasch (see column 1 for the 3-level Rasch random effect and column 5 for the fixed effect estimate). We also observe that standard error of the fixed-effect Rasch model is larger than that of the 3-level random-effect Rasch model and there were difference in the statistical significant for the two methods (fixed-effect Rasch model and 3-level random-effect Rasch model). For the 3-level random-effect Rasch model, only three items (items 1, 7, 6 and 9) were not significant.

Table 2

Comparison of estimates for three-levels random effect and fixed-effect Rasch model

Items	3-level model random effect Rasch Model				Fixed-effect Rasch Model			
	item difficulty estimate	standard error	t-value	Pvalue	item difficulty estimate	standard error	z-scores	Pvalue
Item 1	-1.54	0.31	0.61	0.542	-1.60	0.51	-3.15	< 0.001
Item 2	-2.87	0.44	-3.48	< 0.001	-2.91	0.69	-4.22	< 0.001
Item 3	-3.31	0.61	-4.67	< 0.001	-3.35	0.80	-4.18	< 0.001
Item 4	-1.70	0.71	-4.63	< 0.001	-1.76	0.52	-3.38	< 0.001
Item 5	-0.69	0.46	-3.73	< 0.001	-0.72	0.46	-1.57	< 0.001
Item 6	-0.58	0.40	-1.71	0.087	-0.61	0.45	-1.34	< 0.001
Item 7	1.28	0.40	-1.47	0.144	1.43	0.49	2.90	< 0.001
Item 8	0.56	0.41	3.13	0.002	0.61	0.46	1.34	< 0.001
Item 9	-0.69	0.39	1.44	0.150	-0.72	0.46	-1.57	< 0.001

Variance Component and Intra-Cluster Correlation Coefficient in percent	
intra-school correlation coefficient	0.1 (0.001)
intra-person correlation coefficient	58.2 (1.099)
intra-item correlation coefficient	41.7 (0.789)

na = not applicable

() Variance component

Note: Item 1 is the reference level

In order to obtain a better sense of the importance of the various levels of analysis, we consider the ratio of each variance component to the total variance. With the data measured at person's-level, it should not come as surprise that the variance component at person level accounts for a major proportion of the variance in the data. Specifically, the school-level accounts for 0.1 percent while the person-level accounts for 58.2 percent of variability. Clearly, ignoring the multilevel nature of this data set will have an adverse consequence on how item difficulty estimate will be interpreted and this could produce erroneous statistical inferences in the one-level Rasch measurement model (see, Table 2).

Table 3

Hausman specification test for multilevel Rasch model with random effect

Items	Item difficulty estimates for two and three level				
	$\hat{\alpha}_{q_2}$	$\hat{\alpha}_{q_3}$	\hat{q}_N	S.E	DIF
item1	0	0	na	na	0.09
item 2	-1.24	-1.32	0.09	0.28	0.18
item 3	-1.66	-1.76	0.10	0.34	0.19
Item 4	-0.14	-0.16	0.01	0.20	0.10
Item 5	0.76	0.86	-0.10	0.16	0.00
Item 6	0.85	0.96	-0.11	0.16	-0.02
Item 7	2.46	2.83	-0.37	0.15	-0.28
Item 8	1.84	2.11	-0.27	0.15	-0.18
Item 9	0.76	0.86	-0.10	0.16	0.00
Item 10	1.36	1.54	-0.19	0.15	-0.10
Chi-square			23.02		
Pvalue			0.006		

Note: Item 1 is the intercept

S.E = Standard error

 $\hat{\alpha}_{q_2}$ = Two-level item difficulty $\hat{\alpha}_{q_3}$ = Three-level item difficulty \hat{q}_N = Difference between two and three –levelS.E = sqrt (diag(var[$\hat{\alpha}_{p_2} - \hat{\alpha}_{p_3}$]))

DIF = Differential Item Functioning

Tables 3 present the Hausman specification test for multilevel Rasch model with random effect. The 2-level item difficulty and the 3-level item difficulty estimates are cited in column 1 and 2 while column 3 shows how the differences between 2-level item difficulty and the 3-level item difficulty (\hat{q}_n). The Hausman tests were statistically significant (see the last two rows of Table 3) and the χ^2 value for using Hausman specification test for multilevel Rasch model was ($\chi^2(9) = 23.02, p = 0.006$). However, the DIF values in Table 3, suggest that items 5 and 9 are better reference items than that of item 1 if you wish to report relative to a reference item. The Hausman test suggests a statistical difference in item difficulty estimates between two- and three-level Rasch models with random effects and estimating the Hausman specification test with no

intercept (no reference item), multilevel framework for hierarchical generalised linear models may be lost in terms of model building.

Discussion

Multilevel analyses have become an accepted statistical technique in the field of education where over the past decade or so (Rice & Jones, 1997) the methods have been developed to explore the relationships between person's ability characteristics and the characteristics of the schools they attend. The purposes of this paper are to (i) estimate item difficulty estimate with or without clustering and (ii) the Hausman test approach to multilevel Rasch model. In a nutshell, this paper provides Rasch users with an alternative way of investigating the fit statistics for multilevel Rasch model using the Hausman test.

In this paper, we estimated the fixed-effect Rasch model on the data was estimated by treating the data as "flat" instead as a hierarchy and the multilevel Rasch model for random-effect by was estimated taking into account the hierarchically structured nature of the data (see Table 2 for details). This illustration is useful because it consider the implications of ignoring the multilevel data structure and provide an answer to the question of what the item difficulty estimate would be if we were to ignore the multilevel character of the IAEA data or any other multilevel data.

Multilevel models may increase the number of assumptions that one has to make about the data. Not only do we have to assume the distribution of the person's ability, we have to include the assumption of the some person's ability distribution if we are estimating the item difficulty estimate if there is no intercept (see, Roberts & Herrington, 2005). This assumption could adversely affect the interpretation of the item difficulty estimate in Rasch by slightly increasing item difficulty parameter for three-level (see column 1 and 5 in Table 2).

Researchers should be aware that multilevel Rasch model random effects are intensive and clustering could be just be a statistical nuisance. For example, if there is no statistical

difference in item difficulty parameter between the two- and three-level Rasch model the standard Rasch software (e.g., WINSTEPS and others Rasch software) may be used to analyse a multilevel Rasch data set but if there are statistical differences, the application provided in this paper may be useful.

The application demonstrated in this study shows the statistical benefits of the Hausman test applied to a multilevel Rasch model. Too often Rasch measurement experts (Uekawa, 2005) do not consider the multilevel character of the data when they frequently analyze item difficulty estimate in Rasch. As we have demonstrated here, this can have effects on the item difficulty estimate obtained from the data. Multilevel models offer a statistical tool that can capture the data structure and thereby produce correct item difficulty estimate or inferences (Steenbergen & Jones, 2002).

However, it is pertinent to conclusion that, one main advantage of multilevel Rasch model is that the model decomposes the variance across different levels of analysis and this will enable educational researchers to assess the importance of each level and how it will be lost by ignoring a particular level. As stated earlier, the multilevel Rasch models with random effect item difficulty estimate describe in this paper may depart from the classical fixed-effect Rasch model (Rasch, 1960) because the classical fixed effect Rasch conceptualized each person and each item to have a fixed measure while the random effect measure and each person to be a random representative of a distribution (Mike Linacre, 2006, personnel email, Raudenbush, et al., 2003).

Acknowledgments

The contribution made by Professor Mike Linacre, Dept. of Occupational Therapy, Faculty of Health Sciences, The University of Sydney is gratefully acknowledged.

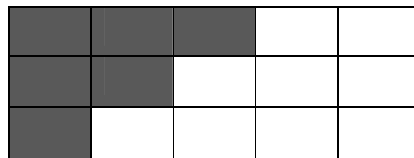
References

- Agho, K.E & Athanasou, J.A. (2005): Demonstration of the bootstrap method of Rasch conditional item difficulty estimation: *Rasch Measurement Transactions* 19.2. Autumn. 1022 – 1023.
- Andersen, E.B. (1973a). A goodness of fit test for the Rasch model. *Psychometrika*, 38, 123-140.
- Andersen, E. B. (1973b). Conditional inference and multiple choice questionnaires. *British Journal of Mathematics and Statistical Psychology*, 26, 31 – 44.
- Beretvas, N.S and Kamata, A. (2005): The Multilevel Measurement Model: Introduction to the Special Issue. *Journal of Applied Measurement*, 6(3), 245-254.
- Berger, J. O. & Sellke, T. (1987). Testing a point null hypothesis: The irreconcilability of p values and evidence (with discussion). *Journal of the American Statistical Association*, 82, 112-139.
- Fisher, G. H. & Molenaar, I.W. (1995). *Rasch models. foundations, recent developments and applications*. New York: Springer-Verlag.
- Hausman, J. A. (1978). Specification tests in econometrics. *Econometrica*, 46(6):1251–1271.
- Linacre, J. M. (2001). *WINSTEPS Rasch measurement computer program (Version 3.31)* [Computer software]. Chicago: Winsteps.com.
- Linacre, J. M. 2006 September 7. Manuscript for consideration [Personal email]. Accessed 2006 September 8.
- Miller, A. J. (1984). Selection of subsets of regression variables (with discussion). *Journal of the Royal statistical Society, Series B*, 147, 389 – 425.
- Pastor, D. A and Beretvas, S. N (2003). *An Illustration of Longitudinal Rasch modeling in the context of Psychotherapy outcomes assessment*. Paper presented at the annual meeting of the National Council of Measurement in education, Chicago, April 21-25.
- Rasch, G. (1960/1980). *Probabilistic models for some intelligence and attainment test*. Chicago: University of Chicago Press. (Originally published by The Danish Institute for Educational Research, Copenhagen, 1960).
- Raudenbush, S.W., Johnson, C and Sampson, R.J. (2003): Multivariate, Multilevel Rasch model with application to Self-reported Criminal behavior. *Journal of sociological methodology*, 33(1), 169-211.
- Rice, N & Jones, A. (1997): Multilevel models and Health Economics. *Journal of Health Economics*, 6(6), 561-75
- Roberts, J.K and Herrington, R. (2005): Demonstration of software Programs for Estimating Multilevel Measurement Model parameters. *Journal of Applied Measurement*, 6(3), 255-272.
- Rosier, M.J. (1980a). *Changes in Secondary School Mathematics in Australia: 1964 to 1978*. Hawthorn: Australian Council for Educational Research.
- Rosier, M.J. (1980b). *Test and Item Statistics for the First and Second IEA Mathematics Studies in Australia*. Hawthorn: Australian Council for Educational Research.
- Skrondal, A. & Rabe-Hesketh, S. (2004). *Generalized latent variable modeling: Multilevel, longitudinal and structural equation models*. London: Chapman & Hall/ CRC.

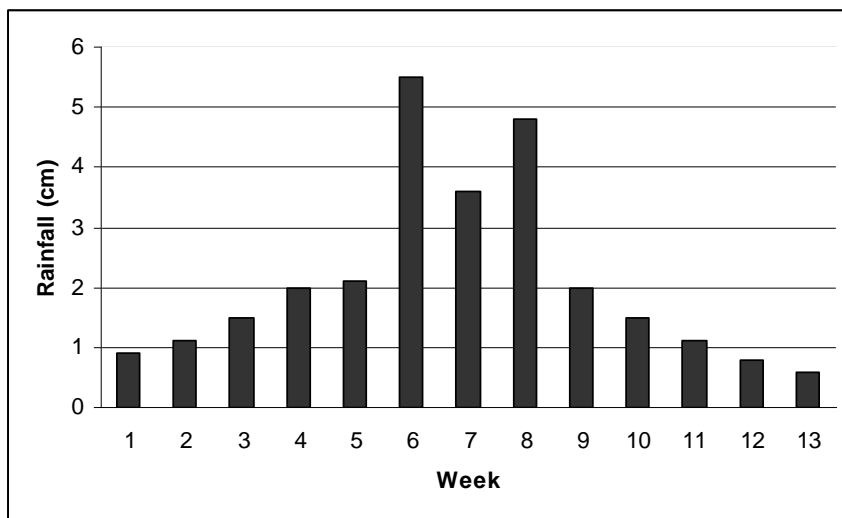
- Steenbergen, M.R., Jones, B.S. (2002). Modeling Multilevel Data Structure. *American Journal of Political Science*. 46(1). 218-237.
- Traub, R.E & Wolfe, R.G (1981). Latent Trait Theories and the Assessment of Educational Achievement. *Review of Research in Education*, 9, 377- 435.
- Uekawa, K (2005):*Why can we do Rasch model using HLM?* Available at <http://www.estat.us/sas/RaschModelEssay.doc>
- Weesie, J. (1999). *The Rasch model in STATA*. STATA statistical software 7.0 : STATA Corporation.

Appendix: First ten questions of the 1978 IAEA mathematics tests

- 1 43.0 - 17.6 is equal to
- 2 How many seven-man teams can you make out of 7 nine-man teams?
- 3 $(22 \times 18) - (47 + 59)$ is equal to
- 4 In the figure below the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to

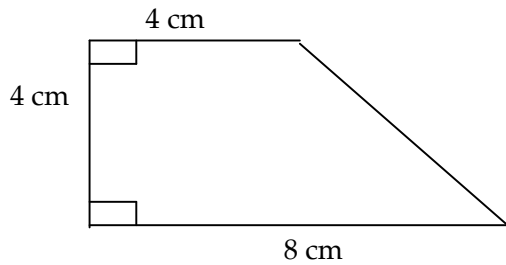


- 5 In the graph below rainfall in cm is plotted for 13 weeks. The average weekly rainfall during the period is approximately?

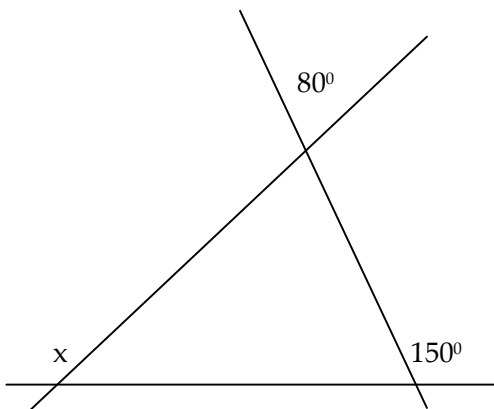


- 6 The value of $2^3 \times 3^2$ is
7. A box has a volume of 100 cm^3 . Another box is twice as long, twice as wide and twice as high. How many cm^3 is the volume of the second box?

8. There is a brass plate of the shape and dimension shown in the figure below. What is its area in square centimeters?



9. What is the square root of 12×75
10. Three straight lines intersect as shown in the figure below. What is x equal to in degrees?



Source of Questions: Rosier, M.J. (1980a). Changes in Secondary School Mathematics in Australia: 1964 to 1978. Hawthorn: Australian Council for Educational Research. Used with permission.